# ACCURACY IMPROVEMENTS IN TWO-PORT NOISE PARAMETER EXTRACTION METHOD

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## ABSTRACT

The 4 transistor noise parameters extraction method based on the measurement of the device noise figure for more than 4 source reflection coefficients and the subsequent minimisation of a suitable error function, have been widely studied. But with the actual improvements in the field of the transistor noise performance( ex HEMT), and the increasing need of accuracy, aiming to obtain reliable models, it becomes necessary to evaluate the precision of the estimated noise parameters.

## INTRODUCTION

In spite of the improvements in the RF instrumentation, network analyser calibration methods and measurement techniques, noise measurements are still more sensitive to errors than other microwave power measurements. The algorithms still used, for the two-port noise parameters determination [1],[2],[3], do not account for measurement uncertainties during the fitting process. This might lead to erroneous values of the noise parameters, far from any physical reality. We have thus introduced a new method, which avoids any linearisation and which fits the best Fmin and Rn, the most sensitive noise parameters. Analytical expression of their variances are given. The optimum reflection coefficient and its module and phase variances are then deduced from a second fit, hence improving its determination. With the aid of the measurement simulation method, we point out the good accuracy obtained, using less reflection coefficient states compared with Mitama's method [4].

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## NEW METHOD

Among the different noise figure formulas of a linear two-port, allowing to use the measured parameters without any change, the one which is function of the reflection coefficient is given below.

$$F = F_{min} + 4\frac{R_n}{Z_o}\frac{|\Gamma_{opt}-\Gamma_s|^2}{|1+\Gamma_{opt}|^2(1-|\Gamma_s|^2)} \quad (1)$$

where F is the measured noise figure, $\Gamma_s$ the measured source reflection coefficient, $F_{min}$ the minimum noise figure, $\Gamma_{opt}$ the optimum reflection coefficient that gives minimum noise figure, $R_n$ the noise equivalent resistance. Zo is a 50 Ohms impedance.
If we set the following change of variables:

$$x_i^j = \frac{|\Gamma_{opt}-\Gamma_{si}^j|^2}{|1+\Gamma_{opt}|^2(1-|\Gamma_{si}^j|^2)} \quad (2)$$

$$y_i^j = F_i^j \quad (3)$$

i=1,..,Ng and j=1,..,Nm

Ng= Number of the source reflection coefficient states
Nm= Number of measurements for the same point.

i is the index of the reflection coefficient being measured, and j is the index of a measurement within Nm measurements done in the same conditions for the same reflection coefficient. We calculate the two statistical means $x_{mi}$, $y_{mi}$ and estimate their corresponding variances $p_i$ and $q_i$, with respect to their systematic and random parts. We assume that $\Gamma_{opt}$ is a constant.

We transpose the paraboloid curve of noise figure from 3 dimensions in the (F,Γ) plane, to a straight line in 2 dimensions in the (Y,X) plane, where $4.R_n/Z_o$ is the slope and $F_{min}$ is the intercept. In this case, we can use the Williamson's algorithm [5], that fits the best straight line by least squares when there are statistical errors in both coordinates.

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In practice, an iterating approach is performed starting with initial values of $R_{n0}$ and $\Gamma_{opt0}$, given by Lane's method [1]. The convergence criterion is set on the slope variation. When it is verified, the estimated value of the intercept is calculated from the following equation:

$$\bar{y} = \beta.\bar{x} + \alpha \qquad (4)$$

Where $\beta$ is the estimated slope, $\alpha$ the estimated intercept and the coordinates have minimum variances. This gives the best precision on the $F_{min}$ determination. Exact expressions are also given for the variances of the slope and intercept in the Williamson's algorithm.

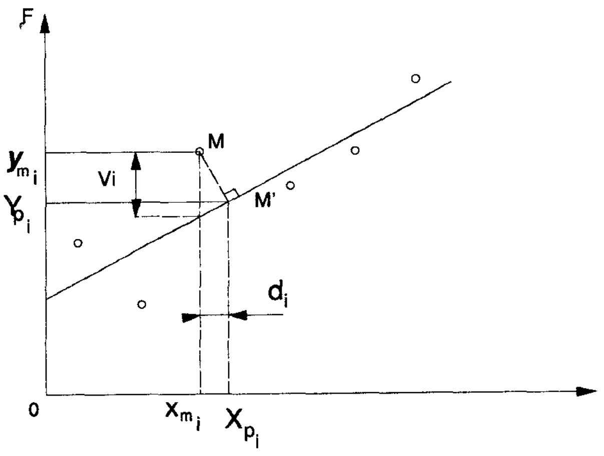


Fig.1 The estimated straight line of the noise figures

The last two noise parameters, $|\Gamma_{opt}|$ and $\Phi_{opt}$, corresponding to the fitted straight line are determined from a second fit. In a first step, an orthogonal transformation is performed. We then calculate the projected point $Xp_i$ on the fitted line, of the measured point $x_{mi}$ using the following expressions:

$$d_i = X_{P_i} - x_{M_i} \quad \text{and} \quad d_i = g_i p_i v_i \beta \qquad (5)$$

The right optimum reflection coefficient is then estimated using the relation below:

$$Xp_i = \frac{|\Gamma_{opt} - \Gamma_{smi}|^2}{|1 + \Gamma_{opt}|^2 (1 - |\Gamma_{smi}|^2)} \qquad (6)$$

We linearise this expression by pointing out the magnitude $\rho_o$, the phase $\Phi_o$ of the optimum reflection coefficient, and $\rho_{smi}$, $\Phi_{smi}$ the means of the source reflection coefficient magnitude and phase.

$$(A_i - 1)\rho_o^2 + 2(A_i + \rho_{smi} \text{COS}(\Phi_{smi}))\rho_o \text{COS}(\Phi_o) + 2\rho_{smi} \text{SIN}(\Phi_{smi})\rho_o \text{SIN}(\Phi_o) = \rho_{smi}^2 - A_i \qquad (7)$$

with $A_i = Xp_i(1 - \rho_{smi}^2)$

In the second step, we apply a least square fitting to the previous over determined system, to estimate the 3 parameters $\rho_o^2$, $\rho_o\text{Cos}(\Phi_o)$, $\rho_o\text{Sin}(\Phi_o)$, and then extract the optimum reflection coefficient with a precisely determined phase. Finally, the variances of $\rho_o$ and $\Phi_o$, can be easily calculated from the variances of $\rho_{smi}$, $\Phi_{smi}$ and $Xp_i$.

## SIMULATION

In order to prove that it is possible to reduce the errors influence on the noise parameters extraction, by choosing the right fitting algorithm, we have developed a simulation software, which allows the test and comparison of the different methods. We start by choosing the true parameters, Fmin=0.37 dB, Rn=27 Ω, |Γopt|=0.71, and Φopt=25°, of a typical HEMT at 4 GHz, and the source reflection coefficients at the transistor input. By referring to the published instrumentation specifications for the systematic error curves [6], taken into account in the program , and appreciation of the random measurement errors; the overall uncertainties are calculated. The program uses random functions to generate the simulated values of $F_i$, $|\Gamma_{si}|$ and $\Phi_{si}$, with respect to the probability distribution functions of each parameter [7], and then extracts the four noise parameters through the different methods. The estimated parameters are then compared with statistical tools, as comparative histograms, RMS errors, and the uncertainty diagrams. We choose to compare our method to Mitama's, one of latest commonly used.

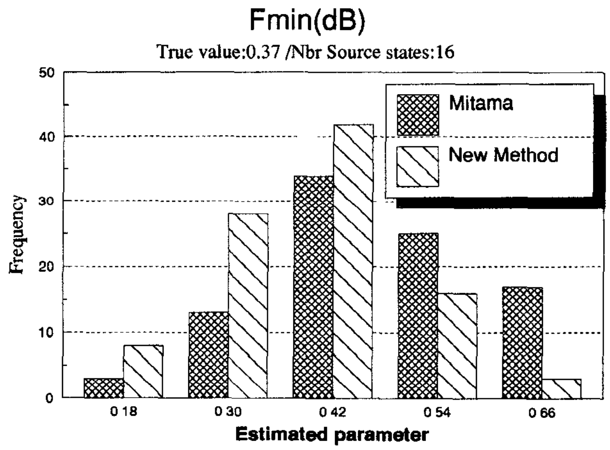


Fig.2

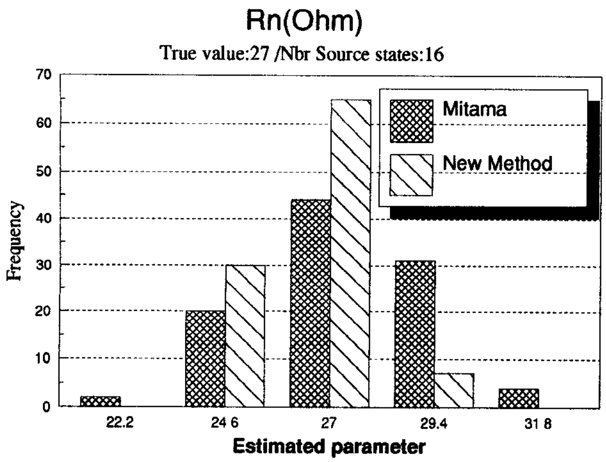


Fig.3

Comparative histograms for 100 simulations

To characterize these methods, we have established a few types of tests, the most significant one being the accuracy test. In this test we calculate the RMS errors, representing the absolute errors mean, which is the difference between the true value and the estimated value of the noise parameter, over 100 simulations. We used this test, with different number of the source reflection coefficient states uniformly and symmetrically distributed on the Smith chart.

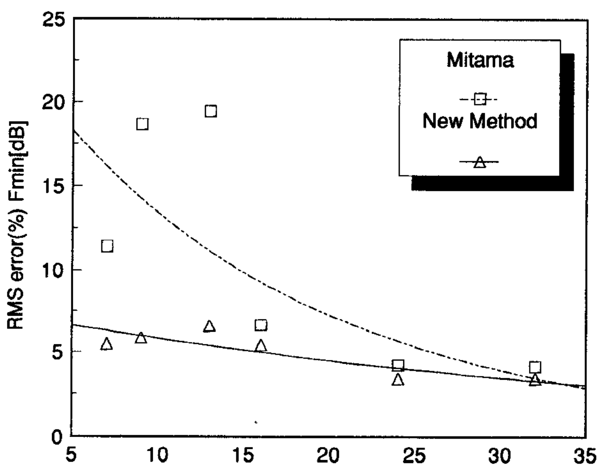


Fig.4

Figures (4) to (7), Noise parameters RMS error in percent of the true value versus the number of source reflection coefficient states.

We point out on figures (4) to (7), that this new method converges, for each noise parameter, more rapidly, for a reduced number of source reflection coefficient states.

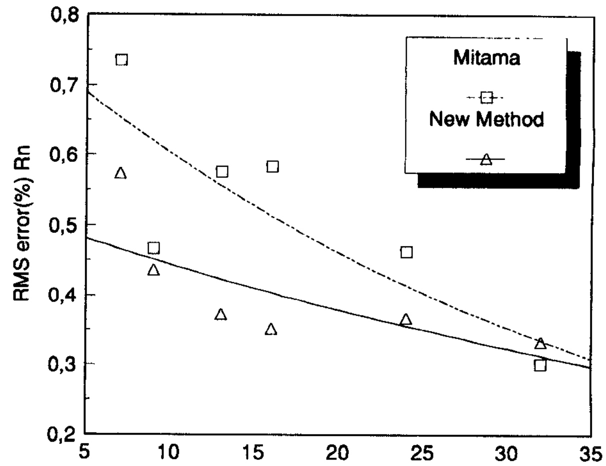


Fig.5

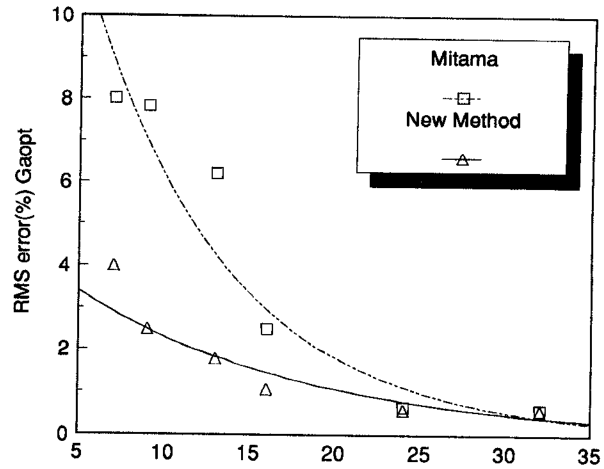


Fig.6

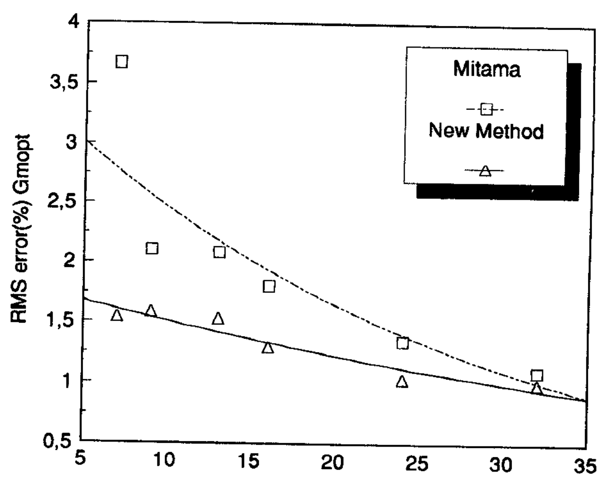


Fig.7

The reason is, by adjusting the best straight line, the algorithm neglects all the points with large variances, which is equivalent to reduce the uncertainty effects. This is a great advantage for fast and accurate automatised noise parameters test set. Another advantage of this new method lies in the possibility to calculate the uncertainties of the estimated parameters. This puts in a prominent position the reliability of the results given by this new method, that will permit the users to proceed to inter-comparisons in a coherent way.

## EXPERIMENTAL RESULTS

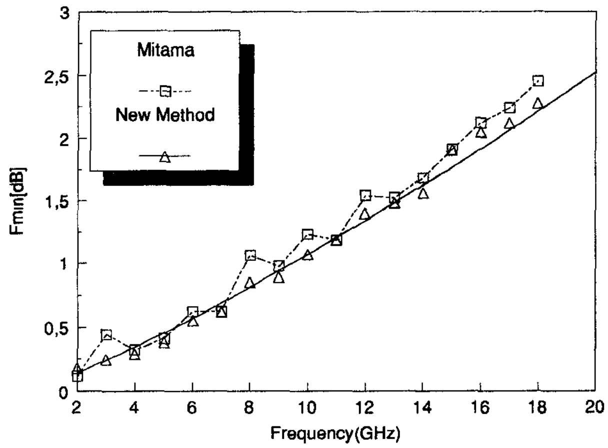


Fig.8

In Figure 8, Measured and smoothed Fmin of a 0.25 micron HEMT. We can notice, that less ripples are obtained with the new method. This indicates that the random errors influence is considerably reduced.

An example of significant accuracy results obtained at 2 GHz with the new method, is shown below:

| | Fmin (dB) | Rn | $\Gamma_{opt}$ | $\Phi_{opt}$ |
|---|---|---|---|---|
| Measured | 0.20 | 21.00 | 0.865 | 10.5 |
| Standard deviation | 0.39 | 0.97 | 0.385 | 2.5 |

## COMPARISON

The simulation results have been compared with the ones given by other authors [8],[9]. The achieved noise parameters error sensitivities through these references, seem to be different. This is understable by various used simulation conditions, especially by $\Gamma_{opt}$ position, and the way to noise the simulated parameters. Nevertheless a first conclusion can be generalized, when $\Gamma_{opt}$ is near 1 on the Smith chart, it is recommended to well distribute the source reflection coefficient states in order to increase the estimation precision.

A comprehensive work about the estimation of noise parameter accuracy have been done in reference [7]. The achieved results confirm that Fmin and Rn are the most sensitive noise parameters, but specially that Fmin uncertainty becomes considerable when $|\Gamma_{opt}|$ increases, what this new method verified with a different approach.

## CONCLUSION

We presented a simple method, for the determination of the two-port noise parameters. It is a weighted least squares regression, where the weights are a function of the measurement uncertainties. This method fits the best Fmin and Rn, which are the most sensitive noise parameters. A more rapid convergence, and a better accuracy are demonstrated with this new method by using a reduced number of reflection coefficient states than with Mitama's method.


## ACKNOWLEDGEMENTS

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